

## 8-1 - 8-3 Notes on Sequences and Series

Sequence - list of terms

Series - sum of a sequence

$n^{\text{th}}$  term of an arithmetic sequence:

1, 3, 5, 7, ...

$$a_n = dn + a_0$$

sum of an arithmetic sequence:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$n^{\text{th}}$  term of a geometric sequence:

1, 2, 4, 8, 16, ...

$$a_n = a_1 r^{n-1}$$

sum of a finite geometric sequence:

$$S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right)$$

sum of an infinite geometric sequence:

$$S = \frac{a_1}{1 - r}$$

Summation notation -

$$\sum_{i=1}^n a_i$$

ending #

expression

Starting #

Ex 1 Find the sum:

$$\sum_{j=3}^8 5j = 15 + 20 + 25 + 30 + 35 + 40$$
$$= 165$$

Ex 2 Find the sum:

$$\sum_{j=0}^6 (j+5) = 5 + 6 + 7 + 8 + 9 + 10 + 11$$
$$= 56$$

Ex 3 Find the sum:

$$\sum_{k=1}^{40} (2k+3) = 5 + 7 + 9 + 11 + \dots + 83$$

arithmetic

$$S_{40} = \frac{40}{2} (5 + 83)$$
$$20 (88) = 1760$$

Ex 4 Find the sum:

$$\sum_{n=1}^{50} (3n - 1) = 2 + 5 + 8 + 11 + \dots + 149$$

$$S_{50} = \frac{50}{2} (2 + 149)$$
$$25(151) = 3775$$

Ex 5 Find the sum:

$$\sum_{i=1}^{10} 2^{i+1} = 4 + 8 + 16 + \dots$$

$$S_{10} = 4 \left( \frac{1 - 2^{10}}{1 - 2} \right) = 4092$$

Ex 6 Find the sum:

$$\sum_{i=1}^{20} 3^{i-1} = 1 + 3 + 9 + \dots$$

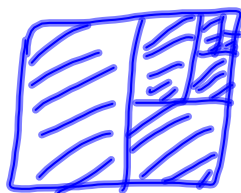
$$S_{20} = 1 \left( \frac{1 - 3^{20}}{1 - 3} \right)$$

$$\approx 1.7 \times 10^9$$

Ex 7 Find the sum:

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

$$S_{\infty} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$



Ex 8 Find the sum:

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

### Homework

p.587 #1-3, 21-23, 43-45, 57-67, 71-73, 79-81 odds

p.598 #1-3, 9-11, 17-19, 27-29, 35-41, 53-55, 61-63,  
67-69 odds

p.607 #1-34 first two problems in each section