8-4 Mathematical Induction

Ex 1 Find a formula for the n[th] term in the sequence:

$n \rightarrow$ 1, 2, 3, 4, ...

$a_n \rightarrow$ 3, 5, 8, 12, 17, 23, . . .

2 3 4 5 6

1 1 1 1

2 steps

$y = ax^2 + bx + c$

$a_n = an^2 + bn + c$

rref $\begin{bmatrix} 1a + 1b + c = 3 \\ 4a + 2b + c = 5 \\ 9a + 3b + c = 8 \end{bmatrix}$

$a = .5$

$b = .5$

$c = 2$

$\boxed{a_n = .5n^2 + .5n + 2}$

Ex 2 Find a formula for the nth term in the sequence:

$x \rightarrow 1 \quad 2 \quad 3 \quad 4 \quad 5$

$y \rightarrow 2, \quad 5, \quad 11, \quad 21, \quad 36, \quad \ldots$

$3 \quad 6 \quad 10 \quad 15$

$3 \quad 4 \quad 5$

$1 \quad 1$

3 steps

$y = ax^3 + bx^2 + cx + d$

$\boxed{a_n = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n + 1}$

$a1 + b1 + c1 + d = 2$

$a2^3 + b2^2 + c2 + d = 5$

$a3^3 + b3^2 + b3 + d = 11$

$a4^3 + b4^2 + b4 + d = 21$

<u>Induction</u> - a method to show a general formula is true

Two steps:

1. Show that it is true for the first term ($n = 1$)
2. Assume that the formula is true when $n = k$ and show that it is true for $n = k + 1$

Ex 3 Prove that $1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2$

① Show that it works for $n = 1$ ✓

$1 = 1^2$

② Assume $1+3+5+7+\ldots+(2k-1)=k^2$ is true

plug in k

Show $\underbrace{1+3+5+7+\ldots+(2k-1)}+(2(k+1)-1) = (k+1)^2$

plug in $k+1$

$k^2+2k+2-1=(k+1)(k+1)$

$k^2+2k+1=k^2+2k+1$ ✓

Ex 4 Prove that $1 + 2 + 3 + 4 + \ldots + n = \dfrac{n(n+1)}{2}$

① $1 = \dfrac{1(1+1)}{2}$ ✓

② Assume: $1+2+3+4\ldots+k = \dfrac{k(k+1)}{2}$

show $\underbrace{1+2+3+4+\cdots+k}+(k+1) = \dfrac{(k+1)((k+1)+1)}{2}$

$\dfrac{k(k+1)}{2}+k+1 = \dfrac{(k+1)(k+2)}{2}$

$\dfrac{k(k+1)}{2}+\dfrac{2k}{2}+\dfrac{2}{2}=\dfrac{(k+1)(k+2)}{2}$

$\dfrac{k(k+1)+2k+2}{2}=\dfrac{(k+1)(k+2)}{2}$

$\dfrac{k^2+k+2k+2}{2} = \dfrac{k^2+k+2k+2}{2}$ ✓

Homework
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#7, 8, 9, 51, 52