

## 8-4 Mathematical Induction

Ex 1 Find a formula for the  $n^{\text{th}}$  term in the sequence:

$$a_n \rightarrow \begin{matrix} n \rightarrow 1 & 2 & 3 & 4 & \dots & 23, & \dots \\ 3, & 5, & 8, & 12, & 17, & 23, & \dots \end{matrix}$$

} 2 steps

$$y = ax^2 + bx + c \qquad a_n = an^2 + bn + c$$

rref

$$\begin{cases} 1a + 1b + c = 3 \\ 4a + 2b + c = 5 \\ 9a + 3b + c = 8 \end{cases}$$

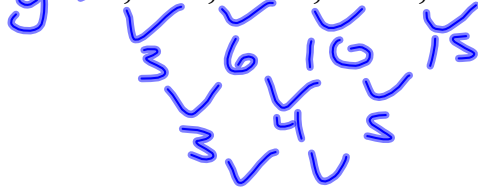
$$\begin{aligned} a &= .5 \\ b &= .5 \\ c &= 2 \end{aligned}$$

$$a_n = .5n^2 + .5n + 2$$

Ex 2

Find a formula for the  $n^{\text{th}}$  term in the sequence:

$x \rightarrow 1 \quad 2 \quad 3 \quad 4 \quad 5$   
 $y \rightarrow 2, \quad 5, \quad 11, \quad 21, \quad 36, \quad \dots$



} 3 steps

$$y = ax^3 + bx^2 + cx + d$$

$$a_n = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n + 1$$

$$a + b + c + d = 2$$

$$a2^3 + b2^2 + c2 + d = 5$$

$$a3^3 + b3^2 + c3 + d = 11$$

$$a4^3 + b4^2 + c4 + d = 21$$

Induction - a method to show a general formula is true

Two steps:

1. Show that it is true for the first term ( $n = 1$ )
2. Assume that the formula is true when  $n = k$  and show that it is true for  $n = k + 1$

Ex 3 Prove that  $1 + 3 + 5 + 7 + \dots + (2n - 1) = n^2$

① Show that it works for  $n = 1$  ✓

$$1 = 1^2$$

② Assume  $1 + 3 + 5 + 7 + \dots + (2k - 1) = k^2$   
*plug in k* *is true*

Show  $1 + 3 + 5 + 7 + \dots + (2k - 1) + (2(k+1) - 1) =$   
*plug in k+1*  $(k+1)^2$

$$k^2 + 2k + 2 - 1 = (k+1)(k+1)$$

$$k^2 + 2k + 1 = k^2 + 2k + 1 \quad \checkmark$$

Ex 4 Prove that  $1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$

①  $1 = \frac{1(1+1)}{2}$  ✓

② Assume:  $1 + 2 + 3 + 4 + \dots + k = \frac{k(k+1)}{2}$

Show  $1 + 2 + 3 + 4 + \dots + k + (k+1) = \frac{(k+1)(k+1+1)}{2}$

$$\frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$$

$$\frac{k(k+1)}{2} + \frac{2k}{2} + \frac{2}{2} = \frac{(k+1)(k+2)}{2}$$

$$\frac{k(k+1) + 2k + 2}{2} = \frac{(k+1)(k+2)}{2}$$

$$\frac{k^2 + k + 2k + 2}{2} = \frac{k^2 + k + 2k + 2}{2} \quad \checkmark$$

Homework  
p.617  
#7, 8, 9, 51, 52