$2+4+6+8+\cdots+2n=n(n+1)$

① $2=1(1+1)$ ✓

② Assume: $2+4+6+8+\cdots+2k=k(k+1)$

Show that $\underbrace{2+4+6+8+\cdots+2k}+2(k+1)=(k+1)(k+1+1)$

$k(k+1)+2(k+1)=(k+1)(k+1+1)$

$k^2+k+2k+2=k^2+2k+k+2$ $(k+2)$

$k^2+3k+2=k^2+3k+2$ ✓

8-5 BINOMIAL THEOREM

$$(x+y)^n = x^n + nx^{n-1}y + \ldots + {}_nC_r x^{n-r}y^r + \ldots + nxy^{n-1} + y^n$$

$(x+y)^0 = 1$

$(x+y)^1 = x + y$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x+y)^5 =$

${}_nC_r$ is the combination of n objects taken r at a time.

$${}_nC_r = \frac{n!}{(n-r)!\,r!} = \binom{n}{r}$$

Ex 1 Find ${}_5C_3$ $= \dfrac{5!}{2!\,3!} = \dfrac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(\cancel{2} \cdot 1)(\cancel{3} \cdot \cancel{2} \cdot \cancel{1})}$

$= 10$

Blaise Pascal
1623 – 1662

Blaise Pascal
1623-1662 Clermont, France

Born on June 19, 1623 in Clermont, France, Blaise Pascal was the son of Etienne Pascal. Blaise's mother died when he was only 3 years onld. Blaise's curiosity rose and he started to work on geometry. By age 16 he had written a number of geometry theorems. Shortly after, Blaise published his first work, *Essay on Conic Sections*. In 1645, Blaise created the Pascaline. By 1647 he had proved the existence of a vacuum and studied pressure changes in the atmosphere. Blaise then published *Treatise on the Arithmetical Triangle*, also known as Pascal's Triangle. In 1654, Blaise laid the foundation for the theory of probability. Later he decided to pledge his life to Christianity. He then published his famous work *Pensees* in 1654, and his last work, *Cycloid*, in 1658. After a lifetime of pain, Blaise Pascal died on August 19, 1662 in Paris, France.

Major Works:
Theory of Probabilities
Pascal's Triangle

Remember Pascal's Triangle:

0th term

0th row → 1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

→ 1 5 10 10 5 1

Ex 2 Find $_5C_3$ using Pascal's Triangle. 10

(careful of the 0^{th} row and 0^{th} term)

Ex 3 Simplify: $(x+y)^5$

$$y^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

Ex 4 Simplify: $(2x - y)^5$

$$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$$x \to 2x \qquad y \to -y$$

$$(2x)^5 + 5(2x)^4(-y) + 10(2x)^3(-y)^2 + 10(2x)^2(-y)^3 + 5(2x)(-y)^4 + (-y)^5$$

$$32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$$

Ex 5 Simplify: $(3y + 2)^5$

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$x \to 3y$ $\quad$ $y \to 2$

$$(3y)^5 + 5(3y)^4(2) + 10(3y)^3(2)^2 + 10(3y)^2(2)^3 + 5(3y)(2)^4 + (2)^5$$

$$243y^5 + 810y^4 + 1080y^3 + 720y^2 + 240y + 32$$

HOMEWORK
p.624
#1-23 odds