



Apr 30-7:43 AM

9-4 Rotation of Conics

Ex 1 Classify the graph of each equation

- $x^2 - 3y^2 - 4x + 1 = 0$   
hyperbola
- $x^2 + y^2 - 10x + 2y + 22 = 0$   
circle
- $x^2 - 4x - 24y - 4 = 0$   
parabola
- $2x^2 + 3y^2 - 28x - 12y + 104 = 0$   
ellipse

May 8-9:00 AM

General equation of a conic section:  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Discriminant:  $B^2 - 4AC = 0$  parabola  
 $B^2 - 4AC < 0$  ellipse  
 $B^2 - 4AC > 0$  hyperbola

May 8-9:02 AM

Ex 2 Classify each conic:

$$x^2 - 4xy + y^2 + 2x - 3y + 1 = 0$$

$(-4)^2 - 4(1)(1) = 12 > 0 = \text{hyperbola}$

$$4x^2 - xy + y^2 + 2x - 3y + 1 = 0$$

$(-1)^2 - 4(4)(1) = -15 < 0 = \text{ellipse}$

$$x^2 - xy + 4y^2 + 2x - 3y + 1 = 0$$

$(-1)^2 - 4(1)(4) = -15 < 0 = \text{Ellipse}$

$$x^2 - 4xy + 4y^2 + 2x - 3y + 1 = 0$$

$(-4)^2 - 4(1)(4) = 16 - 16 = 0 = \text{Parabola}$

May 8-9:03 AM

The conic rotates through  $\theta$  where:

$$\cot 2\theta = \frac{C-A}{B}$$

$\cot 2\theta = \frac{1-1}{2} = 0$   
 $\frac{1}{2} \cot 2\theta = \frac{1}{2} \cdot 0 = 0$   
 $2\theta = \frac{1}{2} \cot^{-1}(0) = \frac{1}{2} \cdot 90^\circ = 45^\circ$

May 8-9:06 AM

Ex 3 Find the angle of rotation of each:

$$x^2 - 4xy + y^2 + 2x - 3y + 1 = 0$$

$\cot 2\theta = \frac{1-1}{-4} = 0 \rightarrow \tan 2\theta = \text{undefined} \rightarrow 2\theta = 90^\circ$

$$4x^2 - xy + y^2 + 2x - 3y + 1 = 0$$

$\cot 2\theta = \frac{4-1}{-1} = -3 \rightarrow \tan 2\theta = \frac{1}{3} \rightarrow 2\theta = 18^\circ \rightarrow \theta = 9^\circ$

$$x^2 - xy + 4y^2 + 2x - 3y + 1 = 0$$

$\cot 2\theta = \frac{1-4}{-1} = 3 \rightarrow \tan 2\theta = \frac{1}{3} \rightarrow 2\theta = 18^\circ \rightarrow \theta = 9^\circ$

$$x^2 - 4xy + 4y^2 + 2x - 3y + 1 = 0$$

$\cot 2\theta = \frac{1-4}{-4} = \frac{3}{4} \rightarrow \tan 2\theta = \frac{4}{3} \rightarrow 2\theta = 53.1^\circ \rightarrow \theta = 26.6^\circ$

May 8-9:10 AM

Homework  
p.697  
#3-14, 27-34  
Classify each conic and find the angle of rotation.

May 8-9:10 AM