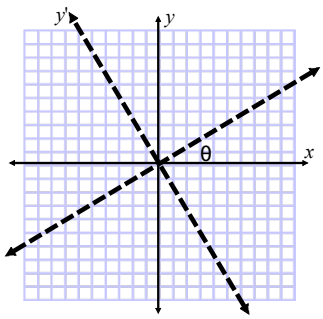
May 5-7:51 AM

9-4 Rotation of Conics (continued)

Step #1

The conic rotates through θ where:

$\cot(2\theta) = \frac{A-C}{B}$



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$\Rightarrow Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

rewrite as $A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$

1. by rotating using $\cot(2\theta) = \frac{A-C}{B}$
2. and using the substitutions $x = x'\cos\theta - y'\sin\theta$, $y = x'\sin\theta + y'\cos\theta$
3. Substitute back into Original equation
4. Simplify into Standard Form

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Ex 1 Write in standard form and graph: $xy - 4 = 0$

$B^2 - 4AC \Rightarrow 1^2 - 4\cdot0\cdot0 = 1 > 0$ hyperbola

$\cot 2\theta = \frac{0}{1}$ $\theta = 45°$

$x = \left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right)$ $y = \left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)$

$\left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right)\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) - 4 = 0$

$\frac{2}{4}x'^2 + \frac{2}{4}x'y' - \frac{2}{4}x'y' - \frac{2}{4}y'^2 - 4 = 0$

$\frac{\frac{1}{2}x'^2}{4} - \frac{\frac{1}{2}y'^2}{4} = \frac{4}{4}$

$\frac{x'^2}{8} - \frac{y'^2}{8} = 1$

C: (0,0)

V: $(\sqrt{8}, 0)$ $(-\sqrt{8}, 0)$

A: $y = \pm\frac{1}{1}(x)$

$y = x$

$y = -x$

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Ex 2 Write in standard form and graph:

$5x^2 - 2xy + 5y^2 - 12 = 0$

$B^2 - 4AC = -2^2 - 4\cdot5\cdot5 < 0$ → ellipse

$\cot 2\theta = \frac{5-5}{-2} = \frac{0}{-2}$ $\theta = 45°$

$x = \left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right)$ $y = \left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)$

$5\left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right)\left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right) - 2\left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right)\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)$

$+5\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) - 12 = 0$

$5\left(\frac{1}{2}x'^2 - 1x'y' + \frac{1}{2}y'^2\right) - 2\left(\frac{1}{2}x'^2 - \frac{1}{2}y'^2\right)$

$+5\left(\frac{1}{2}x'^2 + 1x'y' + \frac{1}{2}y'^2\right) - 12 = 0$

$\frac{5}{2}x'^2 - 5x'y' + \frac{5}{2}y'^2 - 1x'^2 + 1y'^2$

$+\frac{5}{2}x'^2 + 5x'y' + \frac{5}{2}y'^2 - 12 = 0$

$\frac{4x'^2}{12} + \frac{6y'^2}{12} = \frac{12}{12}$ C: (0,0)

$\frac{x'^2}{3} + \frac{y'^2}{2} = 1$ F: (1,0) (-1,0)

$x \to \sqrt{3}$

$y \to \sqrt{2}$

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Homework
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