

9-4 Rotation of Conics (continued)

Ex 1 Write in standard form.

$$x^2 + 2xy + y^2 - \sqrt{2}y = 0 \quad \text{Parabola}$$

$$\cos 2\theta = \frac{1-1}{2} = 0 \quad \theta = 45^\circ$$

$$x = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' \quad y = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$$

$$\left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right)^2 + 2\left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right)\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) + \left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)^2 - \sqrt{2}\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) = 0$$

$$\frac{1}{2}x'^2 - xy' + \frac{1}{2}y'^2 + x'^2 - y'^2 + \frac{1}{2}x'^2 + xy' + \frac{1}{2}y'^2 - x - y = 0$$

$$2x'^2 - x' - y' = 0$$

$$y' = 2\left(x'^2 - \frac{1}{2}x' + \frac{1}{16}\right)$$

$$y' = 2\left(x' - \frac{1}{4}\right)^2 - \frac{1}{8}$$

Ex 2 Write in standard form.

$$13x^2 + 6\sqrt{3}xy + 7y^2 - 16 = 0$$

$$B^2 - 4AC < 0 \therefore \text{ellipse}$$

$$\cos 2\theta = \frac{B}{A-C} = \frac{6\sqrt{3}}{13-7} = \frac{6\sqrt{3}}{6} = \sqrt{3} \quad \tan 2\theta = \sqrt{3}$$

$$2\theta = 60^\circ$$

$$\theta = 30^\circ$$

$$x = \frac{\sqrt{3}}{2}x' - \frac{1}{2}y'$$

$$\text{OR } \frac{\sqrt{3}x' - y'}{2}$$

$$y = \frac{1}{2}x' + \frac{\sqrt{3}}{2}y'$$

$$\text{OR } \frac{x' + \sqrt{3}y'}{2}$$

$$13\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 + 6\sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2}\right)\left(\frac{x' + \sqrt{3}y'}{2}\right) + 7\left(\frac{x' + \sqrt{3}y'}{2}\right)^2 - 16 = 0$$

$$13\left(\frac{3x'^2}{4} - \frac{\sqrt{3}x'y'}{4} - \frac{\sqrt{3}x'y'}{4} + \frac{y'^2}{4}\right)$$

$$+ 6\sqrt{3}\left(\frac{\sqrt{3}x'^2}{4} - \frac{x'y'}{4} + \frac{3x'y'}{4} - \frac{\sqrt{3}y'^2}{4}\right)$$

$$+ 7\left(\frac{x'^2}{4} + \frac{\sqrt{3}x'y'}{4} + \frac{\sqrt{3}x'y'}{4} + \frac{3y'^2}{4}\right) - 16 = 0$$

$$\frac{39x'^2 - 26\sqrt{3}x'y' + 13y'^2 + 18x'^2 + 12\sqrt{3}x'y' - 18y'^2}{4}$$

$$+ \frac{7x'^2 + 14\sqrt{3}x'y' + 21y'^2}{4} - 16 = 0$$

$$\frac{64x'^2 + 16y'^2}{4} = 16 \rightarrow 16x'^2 + 4y'^2 = 16$$

$$\frac{x'^2}{1} + \frac{y'^2}{4} = 1$$

Homework

p.697

#9 and #11