5-5 Notes on Double Angle Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

$\sin(u+v)$

$\sin u \cos v + \cos v \sin u$

$\sin 2u = 2 \sin u \cos u$

- $\sin 2u = 2\sin u \cos u$
- $\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
- $\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

Ex.1 Solve in the interval $[0, 2\pi)$

$2\cos x + \sin 2x = 0$

$2\cos x + 2\sin x\cos x = 0$

$2\cos x(1 + \sin x) = 0$

$2\cos x = 0$ $\quad$ $1 + \sin x = 0$

$\cos x = 0$ $\quad$ $\sin x = -1$

$x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Ex.2 Solve in the interval $[0, 2\pi)$

$\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x + 1 = 0$

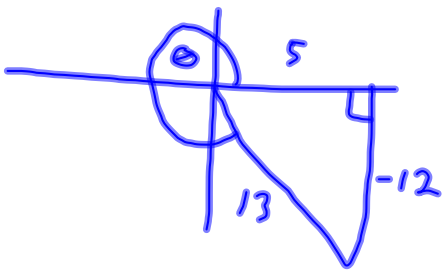
$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -1$

$x = \left\{\frac{\pi}{3}, \frac{5\pi}{3}, \pi\right\}$

$2y^2 + y - 1 = 0$

$(2y - 1)(y + 1)$

Ex.3 Given: $\cos\theta = \frac{5}{13}$ $\quad$ $\frac{3\pi}{2} < \theta < 2\pi$



Find: $\sin(2\theta)$

$2\sin\theta \cdot \cos\theta$

$\frac{2}{1} \cdot \frac{-12}{13} \cdot \frac{5}{13} = \frac{-120}{169}$

$\cos(2\theta)$

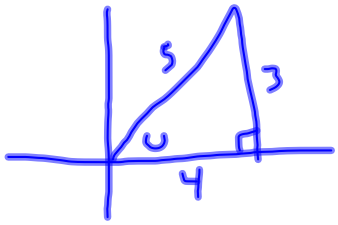
$\cos^2\theta - \sin^2\theta$

$\left(\frac{5}{13}\right)^2 - \left(\frac{-12}{13}\right)^2 = \frac{25}{169} - \frac{144}{169} = \frac{-119}{169}$

$\tan(2\theta)$

$= \frac{120}{119}$

Ex.4 Given: $\sin u = \frac{3}{5}$ $0 < u < \frac{\pi}{2}$



Find: sin (2u)

$2\sin u \cos u$

$\frac{2}{1} \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$

cos (2u)

$\cos^2 u - \sin^2 u$

$\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

tan (2u)

$= \frac{24}{7}$

Homework
p.394
#1-2 & 13-15 all