

## 5-5 Notes on Power-Reducing and Half-Angle Formulas

$$\underline{\cos 2u} = \cos^2 u - \sin^2 u = \underline{2\cos^2 u - 1} = 1 - 2\sin^2 u$$

$$\cos 2v = 2\underline{\cos^2 v} - 1$$

$$\frac{\cos 2v + 1}{2} = \cos^2 v$$

## Power reducing formulas:

$$\sin^2 u = \frac{1 - \cos 2u}{2} \quad \leftarrow$$

$$\cos^2 u = \frac{1 + \cos 2u}{2} \quad \leftarrow$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

$$\cos^2 v = \frac{1 + \cos 2v}{2}$$

$$\cos v = \pm \sqrt{\frac{1 + \cos 2v}{2}}$$

$$\cos \frac{v}{2} = \pm \sqrt{\frac{1 + \cos v}{2}}$$

## Half-Angle Formulas:

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

The + or - sign depends on the quadrant of the angle

Ex.1 Find  $\sin 105^\circ$

$$\begin{aligned} \sin \frac{210^\circ}{2} &= \pm \sqrt{\frac{1 - \cos 210^\circ}{2}} \\ &= \pm \sqrt{\frac{\frac{3}{2} + \frac{\sqrt{3}}{2}}{2}} \\ &= \pm \sqrt{\frac{2 + \sqrt{3}}{4}} \\ &= \pm \frac{\sqrt{2 + \sqrt{3}}}{2} \end{aligned}$$

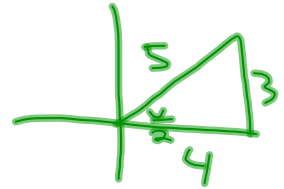
$$\frac{\sqrt{2 + \sqrt{3}}}{2}$$

Ex. 2 Find  $\cos 15^\circ$

$$\begin{aligned} \cos \frac{30^\circ}{2} &= \pm \sqrt{\frac{1 + \cos 30^\circ}{2}} \\ &= \sqrt{\frac{\frac{3}{2} + \frac{\sqrt{3}}{2}}{2}} \\ &= \sqrt{\frac{2 + \sqrt{3}}{4}} \\ &= \frac{\sqrt{2 + \sqrt{3}}}{2} \end{aligned}$$

Ex 3 Find the exact values given:  $\cos u = \frac{7}{25}$

$$0 < u < \frac{\pi}{2}$$



$$\sin \frac{u}{2}$$

$$= \pm \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{\frac{25}{25} - \frac{7}{25}}{2}}$$

$$\cos \frac{u}{2}$$

$$\boxed{\frac{4}{5}}$$

$$\sqrt{\frac{\frac{18}{25}}{2}} = \sqrt{\frac{18}{50}}$$

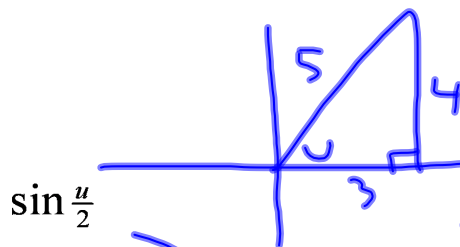
$$\tan \frac{u}{2}$$

$$\boxed{\frac{3}{4}}$$

$$\sqrt{\frac{9}{25}} = \boxed{\frac{3}{5}}$$

Ex 4 Find the exact values given:  $\sin u = \frac{4}{5}$

$$0 < u < \frac{\pi}{2}$$



$$= \pm \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{\frac{3}{5} - \frac{3}{5}}{2}}$$

$$\cos \frac{u}{2}$$

$$= \pm \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 + \frac{3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{4}{5}}$$

$$\tan \frac{u}{2}$$

$$= \frac{1}{2}$$

$$= \sqrt{\frac{4}{5}} = \sqrt{\frac{4}{5}}$$

$$= \frac{2}{\sqrt{5}}$$

$$= \sqrt{\frac{2}{10}}$$

$$= \sqrt{\frac{1}{5}}$$

$$= \frac{1}{\sqrt{5}}$$

Homework  
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