

3-3 Properties of Logarithms

$$\log_a x = \frac{\log x}{\log a} \quad \text{change of base formula}$$

$$\log(u \cdot v) = \log u + \log v$$

~~$\log(u+v)$~~

$$\log\left(\frac{u}{v}\right) = \log u - \log v$$

$$\log u^n = n \log u$$

Ex. 1 Rewrite as a common logarithm and evaluate:

$$\log_5 20 = \frac{\log 20}{\log 5} \approx 1.86$$

$$\log_3 50 = \frac{\log 50}{\log 3} \approx 3.56$$

$$\frac{\ln 50}{\ln 3}$$

Ex 2 Rewrite each logarithm in terms of $\ln 2$ and $\ln 5$.

$$\ln 10 = \ln(2 \cdot 5) = \ln 2 + \ln 5$$

$$\ln \frac{5}{32} = \ln \left(\frac{5}{2^5} \right) = \ln 5 - \ln 2^5$$

$$\ln 5 - 5 \ln 2$$

Ex. 3 Expand each logarithmic expression:

$$\log \frac{5x^3}{y^2} = \log 5 + \log x^3 - \log y^2$$

$$\log 5 + 3 \log x - 2 \log y$$

$$\log \sqrt{x^2 y} \rightarrow \log (x^2 y)^{\frac{1}{2}} = \frac{1}{2} \log (x^2 y)$$

$$= \frac{1}{2} (\log x^2 + \log y)$$

$$= \frac{1}{2} (2 \log x + \log y) = \boxed{\log x + \frac{1}{2} \log y}$$

★ Ex. 4 Expand the logarithmic expression:

$$\begin{aligned}\log \frac{x^4}{y^2 z^5} &= \log x^4 - (\log y^2 + \log z^5) \\ &= 4 \log x - (2 \log y + 5 \log z) \\ &= 4 \log x - 2 \log y - 5 \log z\end{aligned}$$

Ex. 5 Condense the expression to a single logarithm:

$$2 \ln 8 + 5 \ln x$$

$$\ln 8^2 + \ln x^5$$

$$\ln(8^2 x^5)$$

Ex. 6 Condense the expression to a single logarithm:

$$5 \log x - 2 \log y + 3 \log a$$

$$\log x^5 - \log y^2 + \log a^3$$

$$\log \left(\frac{x^5 a^3}{y^2} \right)$$

Homework

p. 211

1-19, 37-47, 59-69

odds