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$\log 5^{\left(-\frac{t}{2}\right)} = \log 0.20$ $\qquad 5^{-1} = \frac{1}{5}$

$-\frac{t}{2} \log 5 = \log 0.20$

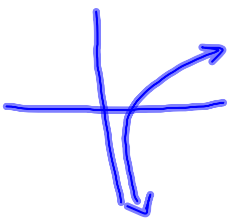
$\cancel{2 \cdot} -\frac{t}{2} = \frac{\log 0.20}{\log 5} \cdot 2$

$t = 2$

Solve: $\log(x + 4) + \log(x + 1) = 1$

$\log 5 + \log 2$

$\cancel{\log -2 + \log -5}$



$\log_{10}\left((x+4)(x+1)\right) = 1$

$10^1 = x^2 + 5x + 4$

$0 = x^2 + 5x - 6$

$0 = (x-1)(x+6)$

$x = \{1, \cancel{-6}\}$

3-5 Exponential and Logarithmic Models

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$ Amount of an investment compounded n times per year

$$A = Pe^{rt}$$ Amount of an investment compounded continuously

Ex.1 If you deposit $5000 in an account earning 8% interest compounded quarterly, how much will you have after 10 years?

$$A = 5000\left(1 + \frac{.08}{4}\right)^{\underline{10 \cdot 4}} = 11{,}040.20$$

Ex.2 In the above example, how long will it take you to double your money?

$$10{,}000 = 5000\left(1 + \frac{.08}{4}\right)^{4t}$$

$$t \approx 8.75 \text{ years}$$

$$\log 2 = \log 1.02^{4t}$$

$$\log 2 = 4t \log 1.02$$

Ex.3 The population of the Earth (in millions) can be modeled by the equation:

$$P = 4450e^{.0173t}$$

where $t = 0$ represents 1980.

What will the population be in 2020?

$$P = 4450e^{(.0173(40))} \approx 8{,}889.8$$

8.889 billion

When will the population reach 11 billion? $\boxed{2032}$

$$11{,}000 = 4450e^{.0173t}$$

$\vdots$

$t \approx 52.3$

Ex.4 Find an exponential growth model whose graph passes through (0, 7.24) and (10, 11.51)

General formula for exponential growth: $y = ae^{bx}$

$7.24 = ae^{b \cdot 0}$

$7.24 = a \cdot 1$

$7.24 = a$

$y = 7.24e^{bx}$

$11.51 = 7.24e^{10b}$

$b \approx .046$

$$\boxed{y = 7.24e^{.046x}}$$

Ex 5 In 1845, Henry David Thoreau built his cabin on Walden Pond for $28.125. The yearly rate of inflation since then is 1.89%. What would be the cost of building this cabin today?

Homework
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