Dave Ebert

Using Statistical Testing to Approximate π

Every March 14, teachers and students around the country celebrate "Pi Day" with a variety of activities. Some students and teachers bake pies, while others plan a special celebration at 1:59 to celebrate Pi Day down to the nearest minute. One of my colleagues gives a prize to the student who can memorize the greatest number of digits of π. They would have to memorize quite a few digits to break the record. In 1995, Hiroyuki Goto memorized 42,194 digits of π. It took him more than nine hours to recite them all.

The activity below offers an examination of π through statistical testing methods for its approximation. Using the definition of π as the ratio of the circumference of a circle to its diameter, high school students should be able to provide evidence that the area of a circle is equal to π times the radius squared—or, in other words, π is equal to the area of a circle divided by the square of its radius. For this activity, we will use circles of radius 1, making the area of the circle equal to π. As outlined in the worksheet, statistics students use their graphing calculators to generate 50 random points with x and y values within the interval [0, 1]. They then draw a circle of radius 1 centered at the origin and plot their 50 random points. By examining the window from (0, 0) to (1, 1), they can calculate the ratio of the points that fall inside the quarter-circle to the total number of random points (50). Multiplying this ratio by 4 approximates the area of the circle, thereby approximating π.

When the class's data are compiled, the mean and standard deviation are calculated, as is a 95 percent confidence interval for the mean. If this procedure were repeated numerous times, 95 percent of the confidence intervals calculated in this manner would contain the true mean. Since this is a small sample, a t-test will be used. The null hypothesis is that the average of all the ratios calculated in this manner is equal to π, or $\mu = \pi$. The alternate hypothesis is that the average is not equal to π, or $\mu \neq \pi$. Students run the hypothesis test and find a value of p. They must

This department is designed to provide in reproducible formats activities appropriate for students in grades 7–12. The material may be reproduced by classroom teachers for use in their own classes. Readers who have developed successful classroom activities are encouraged to submit manuscripts, in a format similar to "Activities" already published. Of particular interest are activities focusing on the Council's curriculum standards, its expanded concept of basic skills, problem solving and applications, and the uses of calculators and computers. Send submissions to "Activities" by accessing **mt.msubmit.net**.

Another source of activities can be found in NCTM's *Using Activities from the "Mathematics Teacher" to Support Principles and Standards*, edited by Kimberley Girard and Margaret Aukshun (order number 12746; $35.95), which also includes a grid to help teachers choose the activities that best meet the needs of their students.

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explain the meaning of the value of p, the probability that the null hypothesis is true.

Students must then list the weaknesses of this method. The majority of mistakes usually involve calculation errors or simple counting errors. Another error could involve points that fall directly on the circle: Should they be counted as occurring inside or outside the circle? This question can be resolved by using the trace function on the calculator and the distance formula to determine precisely whether specific points fall inside or outside the circle.

The final task for students is to repeat this procedure ten times individually. They calculate the mean, the standard deviation of their data, and a 95 percent confidence interval for the mean of their data. They then run a t-test to see if this method does indeed give a good approximation for π. Because the points are selected randomly, we would expect about one 95 percent confidence interval per class not to include π. Likewise, we would expect about one student to reject the null hypothesis.

This method for approximating π incorporating statistical testing can be used in any statistics class. By sharing the connections within mathematics and the history of π with our students, we can make these topics much more interesting and meaningful.

SOLUTIONS

4. The answer approximates π. The radius of the circle is 1, so the area of each quarter-circle is $\pi/4$.

5. Answers will vary, but all should be relatively close to 3.14. Only certain answers are possible, since each student counted the number of points out of 50 and multiplied by 4. Possible answers include 2.72, 2.8, 2.88, 2.96, 3.04, 3.12, 3.2, 3.28, 3.36, 3.44, 3.52, etc.

6. $\bar{x} \approx \pi$. Answers will vary for s.

7. Answers will vary. If this procedure were repeated many times, 95 percent of all confidence intervals thus obtained would contain the true mean.

8. We would use a t-test because we are comparing a mean to a given value and we have fewer than thirty trials.

9. $H_0 : \mu = 3.14$
$H_A : \mu \neq 3.14$

10. The value of p will be a number between 0 and 1, representing the probability that the null hypothesis is true.

11. Answers will vary. This method should give a good approximation of π. Weaknesses of this method may include measurement and/or calculation errors, counting errors, and errors that occur when trying to decide whether points should be counted as being inside or outside the circle.

12. Answers will vary. Individual students should have similar results. In theory, since we are constructing 95 percent confidence intervals, some students in the class may have confidence intervals that do not include π.

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Activity sheet follows.

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Approximation of π

1. Seed the random number generator on your calculator with the last four digits of your telephone number. Type in the four digits, then press STO, MATH, (left arrow), ENTER. Unless two students enter the same number, this procedure will give the entire class different random numbers. See **figure A**.

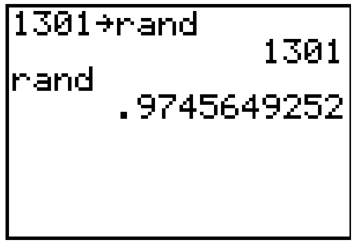

Fig. A

2. Get your calculator ready by setting the window from (0, 0) to (1, 1). See **figure B**. Turn Plot 1 on and choose scatter plot, L_1, L_2, and squares as shown in **figure C**. Clear all equations from the Y= menu.

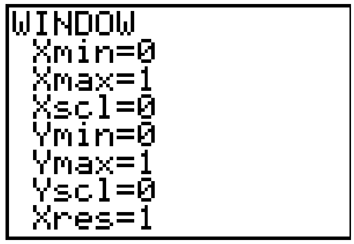

Fig. B

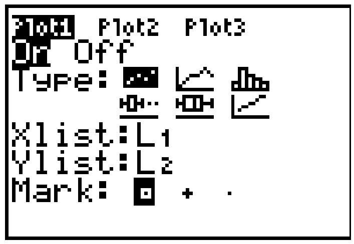

Fig. C

3. Use the three commands shown in **figure D** to generate L_1 and L_2 and to draw a quarter-circle of radius 1. Press 2ND, STAT, (right arrow), 5, to access the Sequence command. Press 2ND, PRGM, 9 to access the Circle command.

   Count the number of points inside the circle. Divide this answer by 50 and multiply the result by 4.

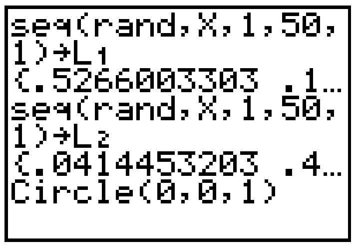

Fig. D

4. What does your answer to question 3 approximate? Explain why this is so.

5. Collect the answers to question 3 from the other members of your class.

6. Find the mean and standard deviation of the class's data.

   $\bar{x}$ = ______________________ s = ______________________

7. Calculate the 95 percent confidence interval for the mean of the class's data. Explain what this means.

Test the claim that this method gives a good approximation of π.

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8. What test will you use? Why?

9. What are the hypotheses?

   H_0 :

   H_A :

10. Using the calculator, find the value of p. Explain what the value of p means.

11. Does this method give a good approximation of π? Describe some of the weaknesses of this method.

12. Repeat the above procedure ten times. Calculate the mean and standard deviation of your data. Then find a 95 percent confidence interval for the mean of your data. Finally, test statistically whether this method gives a good approximation of π. Interpret your results.