City Population, Sunrise/Sunset Project: An Example of an Engaging Activity for Upper-Level Mathematics Students

By David Ebert, Oregon High School, Oregon, WI

In the spring of 2014, the National Council of Teachers of Mathematics released *Principles to Actions: Ensuring Mathematical Success for All.* This publication calls for moving "from 'pockets of excellence' to 'systemic excellence' by providing mathematics education that supports the learning of all students at the highest possible level" (p.3). Eight Mathematics Teaching Practices have been established as a framework to help achieve this vision (Figure 1).

Mathematics reform is needed at all levels to shift school and public perception from mathematics as computation to mathematics as problem solving. To do this, educators and reformers need to start with teacher beliefs set forth in Principles to Actions. Changing teacher beliefs will lead to changing teacher behaviors, for example, attending and learning from professional development opportunities offered through the Wisconsin Mathematics Council (WMC) or Wisconsin Statewide Mathematics Initiative (WSMI). Changing teacher behaviors will lead to changing student beliefs, such as those outlined in Carol Dweck's (2007) *Mindset* or Jo Boaler's (2009) *What's Math Got to Do With It?*. Changing student beliefs will lead to changing student behavior, with the Common Core State Standards (CCSS-M) for Mathematical Practice as an end goal.

The second Mathematics Teaching Practice states that teachers need to implement tasks that promote reasoning and problem solving. "To ensure that students have the opportunity to engage in high-level thinking, teachers must regularly select and implement tasks that promote reasoning and problem solving. These tasks encourage reasoning and access to the mathematics through multiple entry points, including the use of different representations and tools, and they foster the solving of problems through varied solution strategies" (NCTM, 2014, p.17).

What follows is an example of a task used in an upper-level mathematics class that promotes reasoning and problem solving, allows student access through multiple entry points and elicits student responses through multiple solution strategies.

Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Figure 1. Mathematics Teaching Practices from Principles to Actions (NCTM, 2014)

Throughout high school mathematics, students need to be exposed to multiple opportunities to model using mathematics. In fact, Modeling with Mathematics is both a CCSS-M Standard for Mathematical Practice and a Standard for Mathematical Content. The Common Core requires students to "write a function that describes a relationship between two quantities", "distinguish between situations that can be modeled with linear functions and with exponential functions" and "choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline" (NGA/CCSSO, 2010, 70-71).

To start this project, students are asked to select a unique city from around the world. Students list their choices on the board to ensure that every student has a unique city. Then they need to go online to find the population of their city from the years 1910 through 2000. For most cities, students are able to find population data for every ten years, but for some cities the recorded population is more inconsistent. Once the data is collected, the students need to graph the population of their city over time and determine whether a linear, exponential, quadratic, or sine regression model would provide the best fit for the data and would be most useful in predicting the future population of the city. Students then look up the population of their city in 2010, and compare the actual population to the population predicted by their model. To complete this portion of the project, students make a poster that includes a table of the population data, a graph of the population data, the best-fit equation, the curve on the graph, and the predicted population in 2020.

During this portion of the project, students often question which model they should select. They struggle with their desire to find the "right" answer as opposed to having to select the best answer based on the data. They also struggle with using their model to predict a future result, and ask how they can know if their prediction is correct. This leads to a wonderful opportunity to discuss how predictions that mathematicians, social scientists, politicians, and scientists make are people's best guesses based on past results, and the more people understand the data and the mathematics, the better the prediction can be.

For the second part of the project, students are asked to find sunrise and sunset data for their city. They are directed to the website www.timeanddate.com for this information. They need to find the time of the sunrise and sunset, and the length of day (sunlight) in minutes for the first day of every month of the current year. Then they create three separate graphs with this information, plotting the months of the year on the x-axis. Then they discuss some of their observations about the general shape of the graph, the location of the maximum and minimum values, the vertical shift of the graph, the amplitude of the graph, and its period. On their posters, they plot the sunrise, sunset, and length of day graphs, show a sine regression equation, $y = a*\sin(bx+c)+d$, for the length of day, and include an explanation of the amplitude, period, and vertical shift of their graph.

For this portion of the project, students discuss how the location of their city on the globe affects their sine regression equation. Surprisingly, the values of d are relatively consistent. Values of a vary greatly, however, with greater amplitudes for cities farther from the equator. Students also discuss the impact on their graph and equation if a similar city was selected on the opposite side of the Earth. The period of every graph is equal to 12 months, and this investigation helps students understand the nature of periodic data.

To finish the project, students are required to include a picture of the city, a map that shows the location of the city, and an interesting fact about the city. One unique benefit of having students complete this project is the opportunity for students to personalize their work, allowing the teacher to get to know their students better. This past semester, one of my students studied Jaipur, India, because she was adopted from there as an infant. Another student studied Havana, Cuba, because her father emigrated from there and hasn't returned to see his family since. A third student studied Aguascalientes, Mexico, having lived there before moving to the United States when she was five years old. I was able to discover these students' personal stories as a result of assigning this project, and these connections allowed me to get to know my students better and allowed my students to see a personal connection between their lives and the mathematics we were learning (see Figures 2 through 5 for student examples).

Principles to Actions states that "we need to take action to create classrooms and learning environments where students are actively engaged with worthwhile tasks that promote mathematical understanding, problem solving, and reasoning" (NCTM, 2014, p.109). This worthwhile task engages students and promotes mathematical understanding. It is a prime example of the types of engaging tasks we need to implement in our classrooms to foster an environment of problem solving, reasoning, and using mathematics to solve real world problems.

David Ebert, dde@oregonsd.org

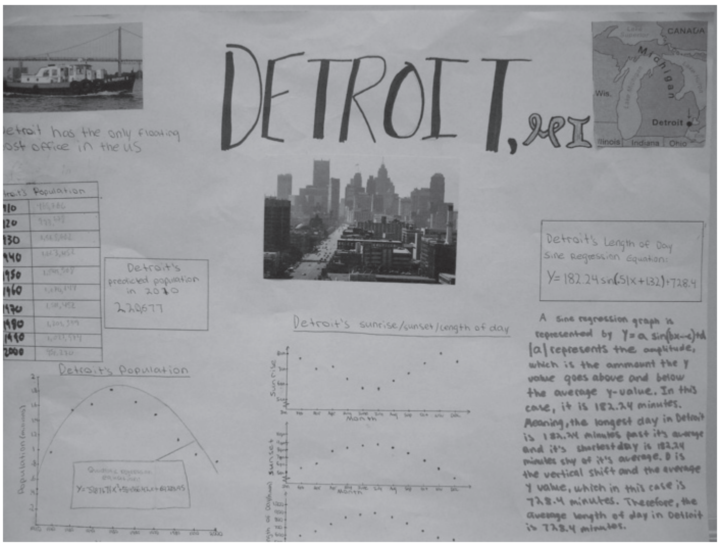

Figure 2. Note the quadratic pattern of the population graph.

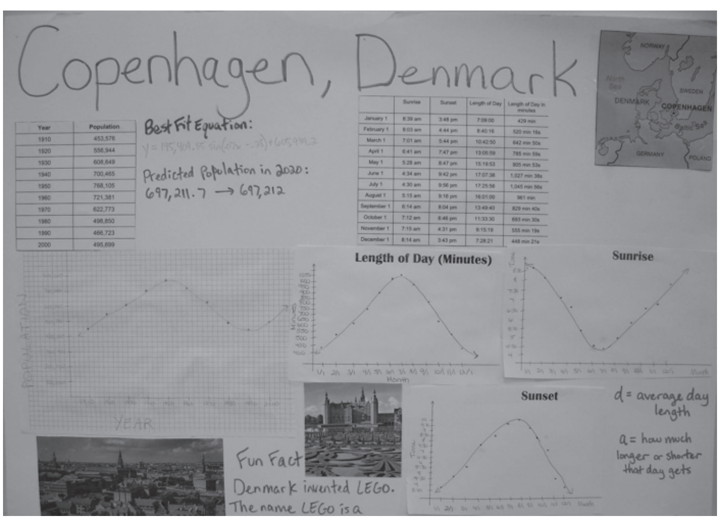

Figure 3. Note the sine curve pattern of the population graph.

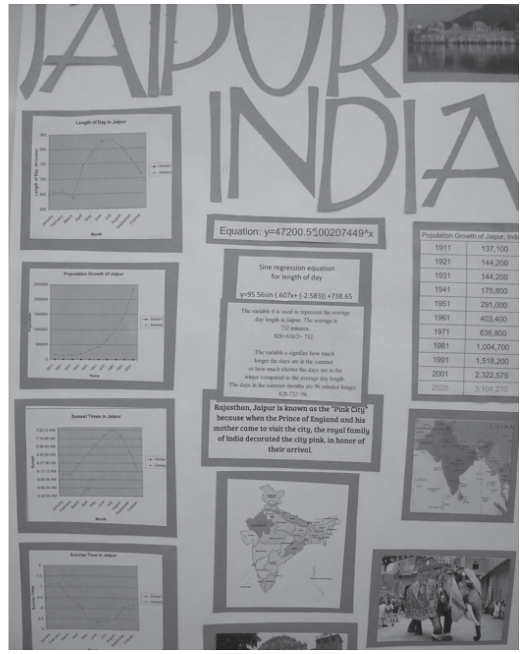

Figure 4. Note the exponential pattern of the population graph.

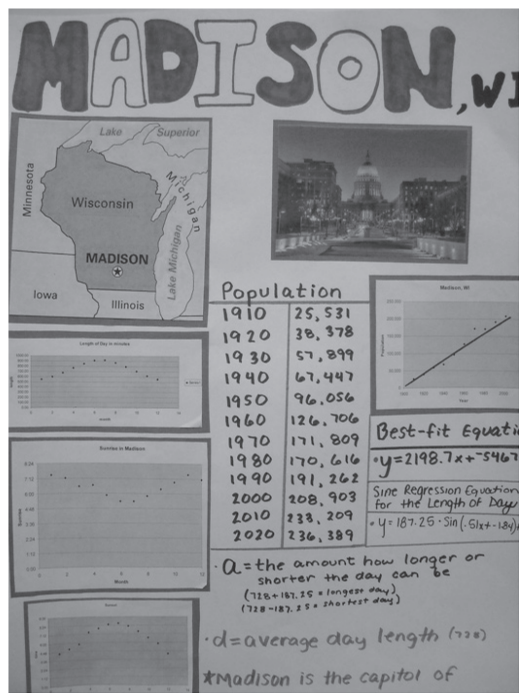

Figure 5. Note the linear pattern of the population graph.

References

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