Engaging Our Students in Upper-level Mathematics: The Power of Parabolas

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In the Spring of 2014, the National Council of Teachers of Mathematics (NCTM) released *Principles to Actions: Ensuring Mathematical Success for All*. This publication calls for moving "from 'pockets of excellence' to 'systemic excellence' by providing mathematics education that supports the learning of all students at the highest possible level" (p. 3). In order to achieve this goal, all teachers must look for ways to engage their students through the discovery of mathematical concepts and the application of mathematics in our world.

When planning a unit, all teachers need to focus on the needs of our students. Do the lessons engage our students? Are our students actively involved in the discovery of new material? Are our students exposed to the applications of the mathematical concepts in other subject areas and/or in the world around us? Teachers can teach any mathematical topic in a teacher-centric manner, or in a student-centric manner that answers all of these questions and leads to student engagement and retention of subject material. This article will focus on ways to engage our students in a unit on parabolas.

The Common Core State Standards for Mathematics (NGA/CCSSO, 2010) includes many examples of content standards related to parabolas that students must know. These content standards, which are included in the Number and Quantity, Algebra, Functions, and Geometry conceptual categories, include:

- Solve quadratic equations with real coefficients that have complex solutions. (N-CN7)
- Factor a quadratic expression to reveal the zeros of the function it defines. (A-SSE3a)
- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A-SSE3b)
- Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. (A-REI4a)
- Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. (A-REI4b)
- Graph linear and quadratic functions and show intercepts, maxima, and minima. (F-IF7a)
- Derive the equation of a parabola given a focus and directrix. (G-GPE2)

Standards on their own do not inspire and excite our students to want to learn more; that is the role of the teacher with a well-designed unit plan. By connecting the mathematics of parabolas to the students' individual experiences, students are intrinsically motivated to extend their experiential and theoretical knowledge of parabolas.

Real-World Applications of Parabolas

Without realizing it, many teachers and students use a three-dimensional paraboloid daily. The cross section of a headlight of a car is a parabola. Rotating the parabola around the axis of symmetry gives a paraboloid. There is a light bulb at the focus, and the paraboloid is a mirrored reflector. Although the light from the bulb disperses in every direction, the parabolic reflector will focus the light in an outgoing beam (Figure 1).

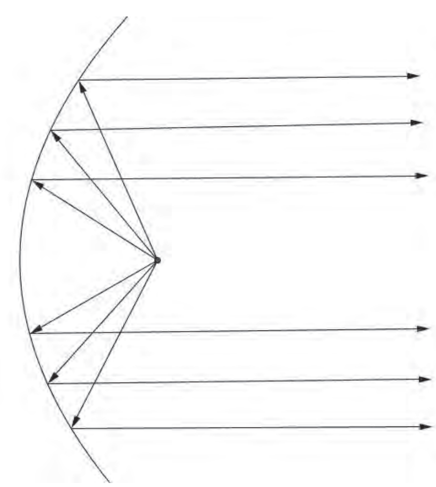

Figure 1. The parabolic reflector within a headlight.

A similar idea works with a satellite dish. Parallel waves are transmitted from a satellite to the dish, focused onto a receiver, and translated into a television signal. With both of these examples, the proper location of the focus is imperative.

Likewise, the Olympic Torch begins its journey every four years by being lit using a parabolic reflector (Figure 2). From Wikipedia:

> The Olympic Torch today is ignited several months before the opening ceremony of the Olympic Games at the site of the ancient Olympics in Olympia, Greece. Eleven women, representing the Vestal Virgins, perform a celebration at the Temple of Hera in which the torch is kindled by the light of the Sun, its rays concentrated by a parabolic mirror. The torch briefly travels around Greece via short relay, and then starts its transfer to the host city after a ceremony in the Panathinaiko Stadium in Athens.

Figure 2. Lighting the Olympic Torch. (source: Wikipedia)

A brief video of the torch lighting can be viewed on YouTube at the following URL: https://youtu.be/U2gbuOGcpzg.

Parabolic solar cookers work in a similar manner. The sun's rays reflect off the parabolic surface to a focus, where water is boiled in a pot or food is cooked in a pan. There is a 15-meter diameter solar bowl in Auroville, India that can cook enough food for 1,000 people per day. At least two humanitarian organizations, the Jewish World Watch and Solar Cookers International, provide parabolic solar cookers to people in need throughout the world.

Parabolic troughs are also used to harness the power of the sun. A parabolic trough is a three dimensional shape with parabolas on two ends and a curved rectangular surface connecting these parabolas. The focus is extended parallel to the curved rectangular shape. Power plants, such as the AREVA Kimberlina Solar Facility in Bakersfield, California, use hundreds of mirrors tilted along a parabolic surface to reflect the sun's rays onto a pipeline that lies at the focus of the parabola. A liquid is pumped through this pipeline, and the reflected solar energy heats the liquid to very high temperatures to generate electricity. A brief video of this solar facility can be viewed on YouTube at the following URL: https://youtu.be/XA9RiNu5ZnI.

Applications in the Classroom

Students learn by doing, so teachers should strive to have their students experience the mathematics they learn to lead to long-term retention. To demonstrate the focus of a paraboloid, obtain an old radar dish and cover it with reflective tape (Figure 3). The author announced to his classes that the first student to bring in an old radar dish would receive a candy bar, and one was brought in the very next day. Once covered, take the radar dish outside on a sunny day. Students can place their hands near to the dish without feeling any heat; once their hands are at the focus, the heat is unbearable. In fact, a dry leaf held at the focus will start to burn.

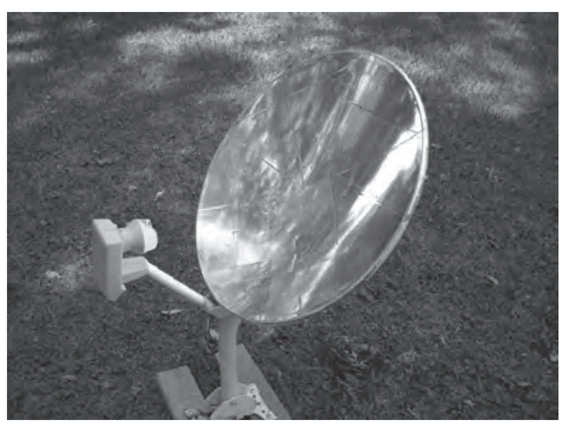

Figure 3. An old radar dish covered with reflective tape.

Another easy way to demonstrate the focus of a paraboloid is with a Fresnel Lens. If one were to take an upward-opening paraboloid and slice it horizontally, then compress those slices onto a flat surface, the result would be a Fresnel Lens. These are relatively inexpensive and are used by farsighted people to magnify text, or by backpackers to light a campfire. Students love using these to light paper on fire, and they demonstrate the focus of a paraboloid very well.

The author has also built a parabolic mini golf hole to demonstrate the focus of a parabola. This was built by drawing a point and a line on a piece of wood, and finding numerous points equidistant from the point (focus) and the line (directrix). These points were connected with a smooth curve, and another piece of wood was cut in this shape. Any golf ball hit parallel to the axis of symmetry will reflect off the parabolic curve and into the hole (Figure 4).

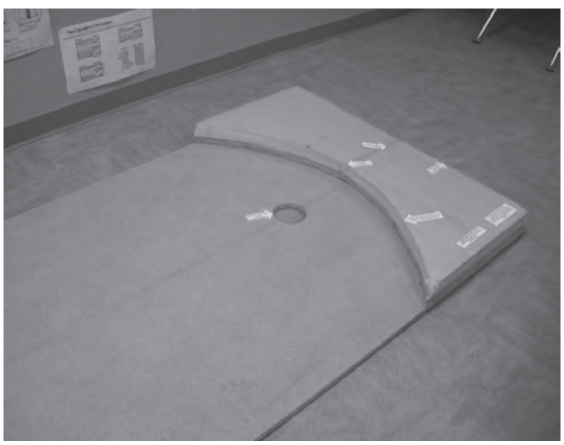

Figure 4. A parabolic mini golf hole.

The best way to have students apply their knowledge about parabolas is to have them build a parabolic hot dog cooker (Figure 5). Given some brief instructions, students can complete this work outside of class time with materials they can find around the house. Here are the steps to build a parabolic hot dog cooker.

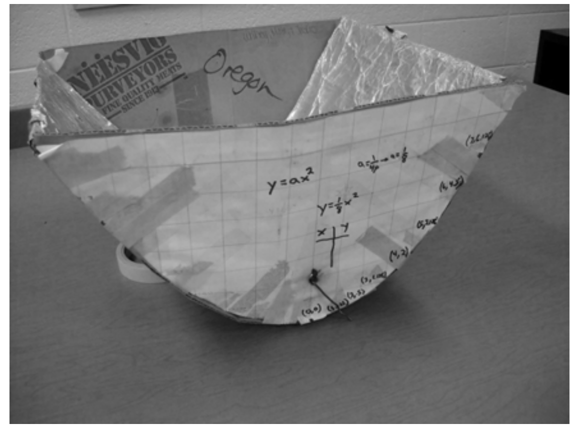

Figure 5. A parabolic hotdog cooker.

1. Determine the distance from the vertex to the focus. Any distance will work, but a distance around two inches works best.
2. Use this distance to find the equation of the parabola. If p is the distance from the vertex to the focus, the equation of a parabola with vertex at the origin is $y = \frac{1}{4p}x^2$.
3. Use the equation of the parabola to generate a table of points, then plot these points on large graph paper, or create a grid on which to plot the points.
4. Glue the graph to cardboard and cut out. Cut out a similar shape for the two ends of the parabolic trough.
5. Cut out a rectangular piece of cardboard using the length of the parabolic piece and a width of about two hot dogs. Score the cardboard by lightly slicing along the grain, and curve to fit between the parabolic pieces. Attach using duct tape.
6. Staple aluminum foil to the curved section.
7. Poke a hole in each parabolic section at the focus. Straighten a metal coat hanger and place it through this hole to use as a skewer for the hot dogs.

Once completed, pick a day to have the students bring in their parabolic hot dog cookers. It is recommended that the class gather in a prominent place so that students can explain the mathematics to any people walking by. Having the teacher provide the hot dogs and the students provide buns, condiments, chips, and beverages gives the perfect opportunity for a class party to celebrate a memorable learning experience.

By connecting the study of parabolas to real-world applications, and having students actively experience the mathematics they learn, students are naturally engaged in their learning. Teaching in a student-centric manner engages all our students and leads to the long-term retention of mathematical concepts.

References

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