

## PARABOLAS INTO PARA-BOO-LAS

In a letter in *Mathematics Teacher* February 2009 (“Reader Reflections,” vol. 102, no. 6, pp. 404–5), I provided some steps for creating a mathematical valentine using a graphing calculator. In *MT* March 2010 (vol. 103, no. 7, p. 470), I gave some guidance for creating a mathematical St. Patrick’s Day card, also using a graphing calculator. My colleague Barb Krueger and I have now collaborated on creating a Halloween card.

The majority of the shapes on the card are created by graphing transformed quadratic equations. Students studying graphing solutions to quadratic equations, translations or reflections, and piecewise functions can develop similar designs based on their own equations. This card is an excellent project for second-year algebra or precalculus students.

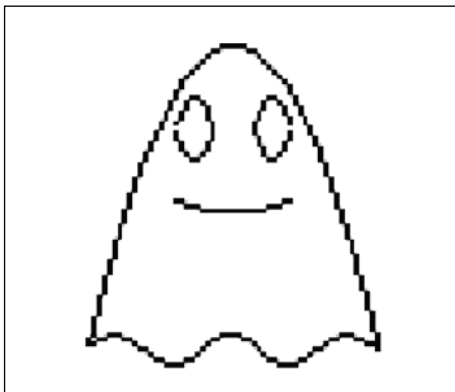


Fig. 1 (Ebert)

We appreciate the interest and value the views of those who write. Readers commenting on articles are encouraged to send copies of their correspondence to the authors. For publication: All letters for publication are acknowledged, but because of the large number submitted, we do not send letters of acceptance or rejection. Letters to be considered for publication should be in MS Word document format and sent to [mt@nctm.org](mailto:mt@nctm.org). Letters should not exceed 250 words and are subject to abridgment. At the end of the letter include your name and affiliation, if any, including zip or postal code and e-mail address, per the style of the section.

The card (fig. 1 [Ebert]) can be folded vertically and horizontally so that the para-boo-la appears on the front of the card and the directions on the inside. I encourage readers to extend this activity and have students develop their own images using transformations. Enjoy—and happy Halloween!

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For a PDF version of the instructions for making this card, go to [www.nctm.org/mt](http://www.nctm.org/mt).

## SUM OF THE ANGLES IN A STAR POLYGON

Regarding star polygons as discussed in “Mathematical Lens” (*MT* February 2008, vol. 101, no. 6, pp. 432–38): **Figure 1 (Wilcock)** shows how a formula can be used to find the sum of the measures of the interior angles of the  $\{7/2\}$  star polygon. The formula can be generalized.

For the  $\{7/2\}$  star polygon, the inscribed angle at vertex  $C$  intercepts the arc  $AE$ , which has measure  $(3 \cdot 2\pi)/7$ . Thus, the measure of angle  $C$  is  $3\pi/7$ . Multiply this result by the number of vertices (7) to obtain the sum of  $3\pi$ .

In general, if the star polygon is a  $\{p/d\}$  polygon, any vertex will intercept

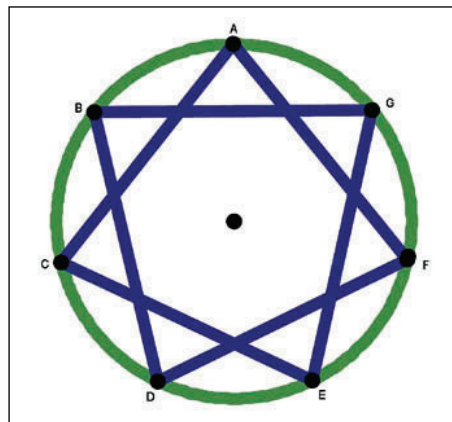


Fig. 1 (Wilcock)

an arc whose length is  $(p - 2d) \cdot 2\pi/p$ . Thus, the inscribed angle at a vertex has measure  $(p - 2d) \cdot \pi/p$ . There are  $p$  vertices, so the sum of the interior angles will be  $(p - 2d) \cdot \pi$ . From this formula, we can see that  $p$  must be greater than  $2d$ . Thus, as shown in table 1, p. 435, when  $d = 2$ ,  $p$  must be at least 5 for the formula to make sense.

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## EXPONENTIAL INEQUALITY

In John Robert Perrin’s article “An Intriguing Exponential Inequality” (*MT* August 2009, vol. 103, no. 1, pp. 50–55), it is shown that the graph of the function  $f(x) = e^{x/e}$  or  $f(x) = (e^{1/e})^x \approx 1.445^x$  has the line with the equation  $y = x$  as a tangent. The proof of this statement can be made somewhat simpler without much calculus.

Let’s consider the exponential function  $f(x) = a^x$ . The derivative at the point  $x = x_0$  is known to be

$$f'(x_0) = a^{x_0} \cdot \ln a,$$

so that the tangent to the graph at the point  $(x_0, f(x_0))$  has the following equation:

$$\begin{aligned} y &= f(x_0) + f'(x_0)(x - x_0) \\ &= a^{x_0} + a^{x_0} \cdot \ln a \cdot (x - x_0) \\ &= a^{x_0} \cdot \ln a \cdot x + a^{x_0} - a^{x_0} \cdot \ln a \cdot x_0 \end{aligned}$$

This equation must be identical with the equation  $y = x$ , and comparison of the slope with the  $y$ -intercept implies that

$$a^{x_0} \cdot \ln a = 1$$

and

$$a^{x_0} - a^{x_0} \cdot \ln a \cdot x_0 = 0.$$

From the latter of these two equations, we can see that