ST. PATRICK'S DAY MESSAGE ON A CALCULATOR

In a reflection in the February 2009 *Mathematics Teacher* (vol. 102, no. 6, pp. 404–5), I provided some steps for creating a mathematical valentine using a graphing calculator. The many positive responses I received regarding that activity have led me to follow it up with a St. Patrick's Day card (see **fig. 1 [Ebert]**), which readers may copy and use in their upper-level mathematics classes. The polar equations in the card have been passed on to me by a number of other creative mathematics teachers.

Although students may create the card just for fun, many mathematical concepts can be derived from this activity. Have students explore the equations used to make the clover shape; then have them fold the card vertically and horizontally so that the clover design appears on the front.

I encourage you to create a set of exploratory questions to go along with this activity. Happy St. Patrick's Day!

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For a PDF version of the instructions for making this card, go to www.nctm.org.

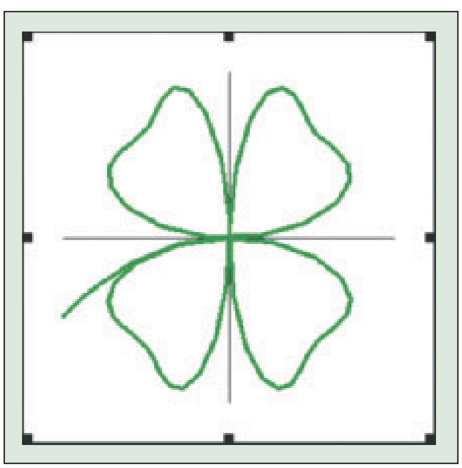

Fig. 1 (Ebert)

A VOLUME GENERALIZATION

In *MT* September 2008 (vol. 102, no. 2, pp. 86–87), the reflection "Egyptian Geometry" deduced the volume formula for a frustum of a square pyramid as

$$V = \left(\frac{h}{3}\right)\left(B^2 + Bb + b^2\right),$$

where h is the altitude and B and b are the lengths of the side of the lower square base and the side of the upper square base, respectively. Actually, we can provide a general volume formula for polygonal prisms, polygonal pyramids, cylinders, cones, frustums of polygonal pyramids, and frustums of cones.

Consider a frustum of an n-sided polygonal pyramid $A_1A_2A_3 \ldots A_n - B_1B_2B_3 \ldots B_n$ (see **fig. 1 [Tu]**). Assume that the original pyramid is $Z - B_1B_2B_3 \ldots B_n$ and that $\overline{ZH}$ is the altitude that intersects the cross-section plane $A_1A_2A_3 \ldots A_n$ at H_1. Since plane $A_1A_2A_3 \ldots A_n \parallel$ plane $B_1B_2B_3 \ldots B_n$, $\overline{ZH}$ is perpendicular to both planes $A_1A_2A_3 \ldots A_n$ and $B_1B_2B_3 \ldots B_n$, implying that $\overline{ZH_1}$ is the altitude of the n-sided polygonal pyramid $Z - A_1A_2A_3 \ldots A_n$ and $\overline{H_1H}$ is the altitude of the frustum of n-sided polygonal pyramid $A_1A_2A_3 \ldots A_n - B_1B_2B_3 \ldots B_n$.

It can be shown that the triangles ZB_1H and ZA_1H_1 are similar. If we let $ZH_1 = u$ and $HH_1 = h$, then

$$\frac{ZB_1}{ZA_1} = \frac{ZH}{ZH_1} = \frac{u+h}{u}.$$

By a similar argument, triangles ZB_1B_2 and ZA_1A_2 are similar, producing the proportion

$$\frac{B_1B_2}{A_1A_2} = \frac{ZB_1}{ZA_1} = \frac{u+h}{u}.$$

By the same reasoning, we can obtain

$$\frac{B_1B_2}{A_1A_2} = \frac{B_2B_3}{A_2A_3} = \cdots = \frac{B_nB_1}{A_nA_1} = \frac{u+h}{u}.$$